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Noise Reduction in support-constrained multi-frame blind-deconvolution restorations as a function of the number of data frames and the support constraint sizes (Preprint)

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Noise reduction in support-constrained multi-frame blind-deconvolution restorations as a function of the number of data frames and the support constraint sizes

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Abstract: We show that the amount of relative noise reduction in multi-frame blind deconvolution image restorations is greatest for just a few data frames and is a more complicated function of the support constraint sizes.

OCIS codes: (100.3020) Image reconstruction-restoration; (100.2980) Image enhancement

1. Introduction

Multi-frame blind deconvolution (MFBD) algorithms seek to estimate jointly an object being imaged along with all the system point spread functions (PSFs) present in the measured data frames. It is well known that the quality of an object restoration improves as the number of data frames included in the restoration process is increased and as the sizes of the support constraints used in the algorithm decrease in size (while still including the true support). This improvement is due to both a greater likelihood of finding the global minimum of the MFBD cost function (when a cost-function based approach is used, of course) and the decreased noise levels in the restored image. In this paper we report on results we have obtained while investigating the latter source of improvement. Our interest in exploring the amount of noise reduction as a function of the number of data frames and the support constraint sizes is due to a desire to better understand the tradeoff between improved image quality and increased algorithm execution time. We show that the amount of total noise reduction in the restored images is an increasing function of the number of data frames, and that the amount of relative noise reduction is greatest when adding including just a few data frames and is greater than might be expected. We define the term "relative noise reduction" in Section 3. We also discuss how the amount of relative noise reduction depends on the object and PSF support constraint sizes. Because we desire to obtain answers that are algorithm independent, we employ a Cramér-Rao lower bound (CRB) approach in the analysis. The outline of this paper is as follows: Section 2 contains a description of the imaging model and CRB theory, results are given in Section 3, and conclusions are presented in Section 4.

2. Imaging model and Cramér-Rao lower bound theory

The equation describing image formation is

$$\mathrm{i}_{\mathrm{m}}(\mathbf{x})=\mathrm{h}_{\mathrm{m}}(\mathbf{x})*\mathrm{o}(\mathbf{x})+\mathrm{n}_{\mathrm{m}}(\mathbf{x}) \quad ; \quad \mathrm{m}=1,\ldots,\mathrm{M} \tag{1}$$

where $*$ denotes convolution, $\mathbf{x}$ is a two-dimensional spatial variable, $i_m(\mathbf{x})$ is the m[th] data frame, $h_m(\mathbf{x})$ is the m[th] PSF, $o(\mathbf{x})$ is the object being imaged, $n_m(\mathbf{x})$ is the m[th] noise realization, and bold-faced type denotes vectors and matrices. Because CRB theory requires a set of random variables, not stochastic processes, Eq.(1) must be rewritten in a vector form rather than as a continuous function. To this end, let α be a square grid of spatial locations of the intensity values of $i_m(\mathbf{x})$ and let $\mathbf{y}_m$, $\boldsymbol{\psi}$, $\boldsymbol{\phi}_m$ and $\boldsymbol{\eta}_m$ be one-dimensional vectors that contain the values of $i_m(\alpha)$, $o(\alpha)$, $h_m(\alpha)$, and $n_m(\alpha)$, respectively, on the grid defined by α. The vectors $\mathbf{y}_m$, $\boldsymbol{\psi}$, $\boldsymbol{\phi}_m$ and $\boldsymbol{\eta}_m$ can be generated from $i_m(\alpha)$, $o(\alpha)$, $h_m(\alpha)$ and $n_m(\alpha)$ by stacking their columns. In addition, let $\mathbf{H}_m$ be the block-circulant matrix associated with $\boldsymbol{\phi}_m$ [1]. Then Eq.(1) can be rewritten as

$$\mathbf{y}_{\mathrm{m}}=\mathbf{H}_{\mathrm{m}}\boldsymbol{\psi}+\boldsymbol{\eta}_{\mathrm{m}} \quad ; \quad \mathrm{m}=1,\ldots,\mathrm{M} \tag{2}$$

Multi-frame blind deconvolution algorithms seek to estimate jointly the parameters contained in the vectors $\boldsymbol{\psi}$ and $\{\boldsymbol{\phi}_m\}$ given the data vectors $\{\mathbf{y}_m\}$, where the quantities in braces are the collection of vectors for all m. Although we could have carried out this analysis using sample statistics generated from restorations produced by a specific MFBD algorithm, we desired to generate algorithm-independent results. Therefore, we chose to use CRB theory [2] for the analysis since it produces lower bounds to the variances of any unbiased estimates of a set of parameters. The CRBs for any unbiased estimates of the elements of the concatenated vector $\boldsymbol{\theta}=\left[\boldsymbol{\psi}^{\mathrm{T}},\boldsymbol{\varphi}_1^{\mathrm{T}},\ldots,\boldsymbol{\varphi}_{\mathrm{M}}^{\mathrm{T}}\right]^{\mathrm{T}}$ are the diagonal elements of the inverse of $\mathbf{F}$, the FIM associated with $\mathbf{y}_m$ and $\boldsymbol{\theta}$. The element of $\mathbf{F}$ in the p[th] row and the q[th] column is given by

$$\mathbf{F}_{\mathrm{pq}}(\boldsymbol{\theta})=\mathrm{E}\left[\frac{\partial\ln \mathrm{f}(\mathbf{y};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_{\mathrm{p}}}\frac{\partial\ln \mathrm{f}(\mathbf{y};\boldsymbol{\theta})}{\partial\boldsymbol{\theta}_{\mathrm{q}}}\right] \tag{3}$$

where $f(\mathbf{y};\theta)$ is the probability density function of $\{\mathbf{y}_m\}$ parameterized by the vector $\boldsymbol{\theta}$ and ln denotes the natural logarithm. Equation (3) gives the elements of $\mathbf{F}$ for the imaging model of Eq.(2) without the application of any constraints. Support constraints are implemented by including in the vectors $\boldsymbol{\psi}$ and $\{\boldsymbol{\phi}_m\}$ only those elements that are inside their support constraint regions [3]. The unbiased CRBs of the parameters $\boldsymbol{\theta}$, CRB($\boldsymbol{\theta}$), are given by

$$\mathrm{CRB}(\boldsymbol{\theta}) = \mathrm{diag}\left(\mathbf{F}^{-1}\right) \tag{4}$$

where diag() denotes a vector containing the diagonal elements of the bracketed matrix. Because of the scaling redundancy of blind deconvolution, this expression for $\mathbf{F}$ is non-invertible. We make $\mathbf{F}$ invertible by estimating only N-1 elements of $\boldsymbol{\psi}$ and N_m-1 elements of $\{\boldsymbol{\phi}_m\}$, where N and N_m are the numbers of pixels in the support constraints applied to these vectors, and by requiring that all the $\mathbf{H}_m$ have full rank, which occurs when the Fourier transforms of $\{h_m(\boldsymbol{\alpha})\}$ are non-zero.

3. Results

We calculated CRBs using the object in Fig.1 for $o(\mathbf{x})$ and zero mean white noise with variance σ^2 for $n_m(\mathbf{x})$. The PSFs $\{h_m(\mathbf{x})\}$ are related to atmospheric PSFs for $D/r_o = 8$, where D is the telescope diameter and r_o is the atmospheric correlation length [4]. We created $\{h_m(\mathbf{x})\}$ by creating $D/r_o = 8$ PSFs, Fourier transforming them, cutting out a square portion of this Fourier transform centered at zero frequency and contained within the telescope OTF support, and then inverse Fourier transforming the results. The resulting $\{h_m(\mathbf{x})\}$ are invertible, making $\mathbf{F}$ invertible. We chose to do this to avoid the complications of calculating and interpreting biased CRBs.

A plot of the unbiased CRBs for $\boldsymbol{\psi}$, $\mathrm{CRB}_M(\boldsymbol{\psi})$, normalized to one for M=1, is given in Fig. 2 as a function of the number of data frames M included in the MFBD process. For this plot, the true object support was used for $\boldsymbol{\psi}$ and a circular support that contained more than 99% of the energy of the $\{h_m(\mathbf{x})\}$ was used. For each M, the corresponding point in the plot is the sum of $\mathrm{CRB}_M(\boldsymbol{\psi})$, denoted sum($\mathrm{CRB}_M(\boldsymbol{\psi})$). Notice that sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) is a decreasing function of M, as expected. Simplistically, one might expect sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) to decrease as 1/M since the noises are statistically independent from frame to frame. This 1/M behavior is present, for example, when using speckle imaging techniques to estimate the energy spectrum of an object [4]. To investigate this expectation, we plotted the function 1/M in Fig. 2 as well. Notice that the sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) plot decreases more rapidly than 1/M for small values of M. We refer to the slope of sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) as the amount of relative noise reduction. This implies that MFBD image restorations benefit more than might be expected from adding just a few frames to the estimation process as compared to carrying out blind deconvolution using only one data frame. We have seen this behavior in restorations obtained using field data as well. Notice also that the slopes of the sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) plot and the 1/M plot appear to be equal for larger values of M. This means that the expected 1/M relative noise reduction in the restored images occurs for larger values of M.

(a) (b) (c) (d) (e)

Fig. 1. Computer-simulated satellite model (a), and support constraints used for CRB calculations: (b) true, (c) blur2, (d) blur 5, and (e) blur 7.

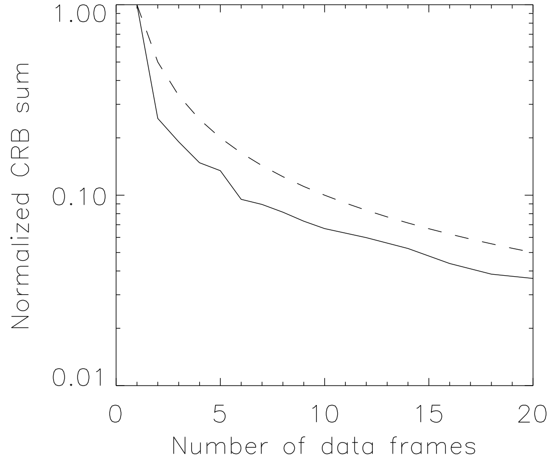

Fig. 2. Plots of the normalized sum($\mathrm{CRB}_M(\boldsymbol{\psi})$) values (solid line) and 1/M (dashed line) as a function of M

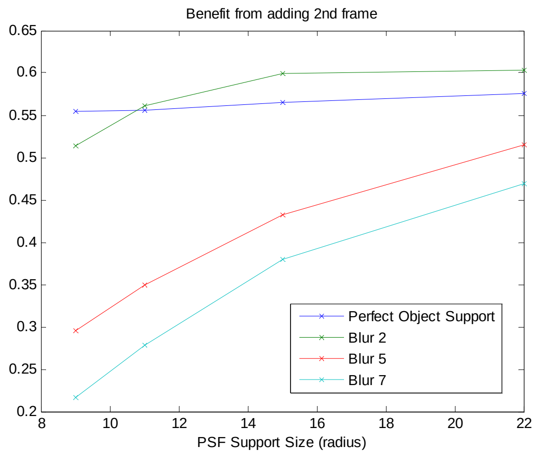

Fig. 3. Decrease in the normalized sum($CRB_2(\boldsymbol{\psi})$) values as a function of object and PSF support sizes.

We then investigated how the amount of relative noise reduction that occurs in sum($CRB_M(\boldsymbol{\psi})$) as a result of using two (M=2) instead of one (M=1) frames of data depends on the object and PSF support constraint sizes. To do this, we calculated sum($CRB_2(\boldsymbol{\psi})$) for several object and PSF support constraints. We used four different object support constraint sizes: the true support region and three larger support regions created by blurring the true support region with 2 x 2, 5 x 5, and 7 x 7 blurring kernels (see Fig. 1). The PSF supports were all circular with increasing radii. The amounts of relative noise reduction are displayed in Fig. 3. Several properties of sum($CRB_M(\boldsymbol{\psi})$) can be observed. The first property is that sum($CRB_2(\boldsymbol{\psi})$) is an increasing function of the PSF support constraint size for all object support constraint sizes, and the rate of increase grows as the size of the object support constraint increases. The second property is that the benefit of adding a second frame to the MFBD process usually increases as the object support constraint size increases for a fixed PSF support constraint size. This second property is especially useful since highly-accurate object supports are difficult to generate, in general.

We emphasize that the results in Fig. 3 are based on the normalized sum($CRB_M(\boldsymbol{\psi})$) values, not the absolute values. Without this awareness, the results in Fig. 3 could be interpreted to mean that a less-accurate object support constraint produces lower CRB values than does a more-accurate object support constraint, which is not true. The proper conclusion to draw from Fig. 3 is that noise reduction occurs more swiftly as M increases for less-accurate support sizes.

4. Conclusions

Using an algorithm-independent CRB approach, we have analyzed the amount of noise reduction possible when using MFBD algorithms. We investigated the amount of absolute and relative noise reduction as a function of the number of data frames included in the restoration process and the sizes of the object and PSF support constraints. We showed that the relative noise reduction is greater than 1/M for values of M on the order of one, where M is the number of data frames, and is approximately equal to 1/M for larger values of M. We also showed that the amount of relative noise reduction achieved for M = 2 is generally an increasing function of object support constraint size. For many object support constraint sizes, the amount of relative noise reduction is a decreasing function of the PSF support size.

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